Incorporating body condition into the analysis of animal movement

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Introduction

What drives an animal's behavior?

The study of animal movement as it reveals insights into animal behavior is of great interest in ecology. Further interests lie in understanding drivers of animal behavior, e.g. impact of habitat, time of day, season.

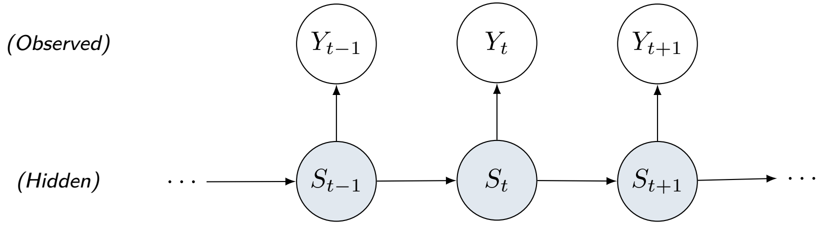
Hidden Markov Models (HMMs)

Animal movement data often takes the form of data collected over time, sometimes at regular temporal scales. Hidden Markov models (HMMs) then provide an intuitive mathematical structure that matches our biological intuition of the movement process – the movements we observe are a product of an unobserved behavioral process.

HMMs

A N-state HMM, for a fixed $N \in \mathbb{N}$, is a doubly stochastic time series model composed of an observation process, $\{\boldsymbol{Y_t}\}$, driven by an underlying latent (hidden) process $\{S_t\}$. In its basic formulation, an HMM is defined by three components:

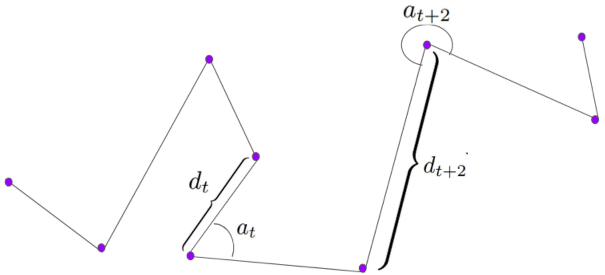
- state-dependent densities $f(\boldsymbol{y_t} \mid S_t = n)$, $n = 1, \ldots, N$
- a transition probability matrix (t.p.m.) $\boldsymbol{\Gamma}$
- an initial state distribution $\boldsymbol{\delta}$



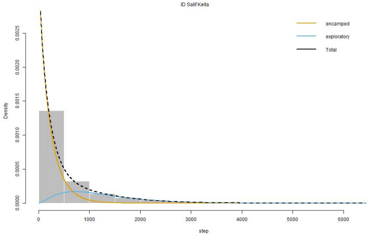
Animal Movement Data: Positions Over Time

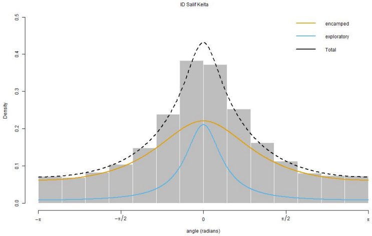
In the context of animal movement, positional data is gathered via GPS or accelerometers, and, in order to connect the positions of the animal to behaviors of interest, this is transformed into step lengths, d_t, and turning angles, a_t, (Morales, 2004).

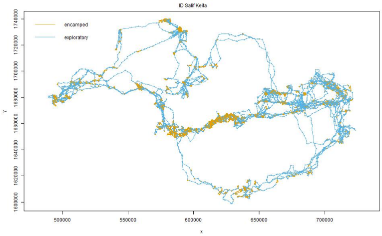
Thus, the observed process results in $\{Y_t\} = \{d_t, a_t\}$.



Data analysis example: African elephant movement







Animal condition

In classical approaches, due to the difficulties to gather observations in a frequent basis, animal's condition is not incorporated as a factor that explains the movement, and thus behavioral, process.

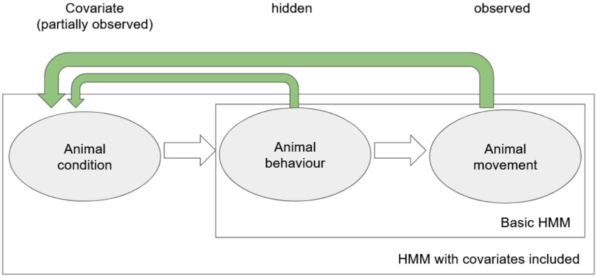
How can the condition process be predicted by incorporating the animal movement data and behavior process?

Research goals

Objective

- Incorporate animal behaviour dynamics and animal movement observations to predict animal's condition,
- Use predictions to explain the animal's movement dynamics

This is known as **feedback mechanisms**



Case study: Sheep movement data

A flock of 60 sheep from northern Patagonia, in Argentina, was monitored using GPS and their location was collected during 8 months, every five minutes. Their body fat percentage were also collected approximately every month.

Model formulation

Data Structure

We construct a general framework to model two types of data sources, positional data and physiological data. Percentage body fat (in kg) in Merino sheep as a proxy for sheep's overall body condition.

- Observations of condition over time t, $\{g_t\}_{t=1}^{T}$ (coarse temporal scale; collected once a month)
- For time t, observations of movement in the interval $[t, t+1)$ distance, $\{d_{t,k}\}_{k=1}^{K}$, and turning angle, $\{a_{t,k}\}_{k=1}^{K}$, $K \in N$ (fine temporal scale; reported every 5 minutes).

Model formulation

Transition probability matrix

- We assume a first-order Markov property for the evolution of the state sequence, so that $S_{t,k}$ depends on other states only through $S_{t,k-1}$, if $k > 1$, or $S_{t-1,K}$ if $k = 1$.
- We allow the condition of an animal, g_t, during the t^{th} interval, $[t, t+1)$, to affect the manner in which the states are generated, by incorporating g_t as a covariate.

We denote the t.p.m. during the t^{th} period as $\mathbf{\Gamma}^t(g_t)$, with entries $\gamma_{ij}^t(g_t) = \Pr(S_{t,k} = j \mid S_{t,k-1} = i, g_t)$, $i, j \in \{1, \ldots, N\}$ and

$$\gamma_{ij}^t(g_t) = \frac{\exp(\rho_{ij}^t)}{\sum_{j=1}^N \exp(\rho_{ij}^t)}, \qquad \text{where} \qquad \rho_{ij}^t = \begin{cases} \tau_0^{(ij)} + \tau_1^{(ij)} g_t & \text{if } i \neq j; \\ 0 & \text{otherwise.} \end{cases}$$

Missing observations of g_t

What do we do if g_t is not observed at time t?

The body fat percentage can be expressed as the ratio of two latent processes:

$$g_t = \overbrace{\underbrace{\frac{b_t}{w_t}}_{\text{overall body mass}}}^{\text{body fat}}$$

where $w_t =$ body fat $+$ lean mass $= b_t + m_t$

Energetic Balance

To describe the evolution of dynamics of g_t, we need to understand how sheep gains/loses energy, and subsequently converts these into gains/losses in body fat and lean mass. Of interest then is the energetic balance, E_t:

$$E_t(\mathbf{S}_{t-1}, \mathbf{d}_{t-1}) = \underbrace{\left[\sum_{k=1}^{K} \mathsf{I}(S_{t-1,k} = \text{foraging})\right]}_{\text{Behaviour process}} \boldsymbol{I} - \underbrace{\left[\sum_{k=1}^{K} d_{t-1,k}\right]}_{\text{Animal movement}} \boldsymbol{L_{t-1}} - \frac{K}{288}\boldsymbol{C_{t-1}}$$

- Energetic intake $\boldsymbol{I}$ (sheep exhibiting foraging behavior)
- Movement (locomotion) costs $\boldsymbol{L_t}$
- Daily maintenance costs $\boldsymbol{C_t}$

Evolution of Body Fat and Lean Mass

The evolution of body fat, b_t, is then (Robbins 1993)

$$b_t = b_{t-1} + \frac{.8 * E_t}{38.12}, \tag{1}$$

and the evolution of muscle mass, m_t, is given by,

$$m_t = m_{t-1} + \frac{.2 * E_t}{22.64}. \tag{2}$$

Since b_t, m_t depends on E_t, a question arises:

How $\sum_{k=1}^{K} \mathsf{I}(S_{t,k} = \text{foraging})$ is distributed?

Distribution computation of $\sum_{k=1}^{K} \mathsf{I}(S_{t,k} = \text{foraging})$

Let's consider the case $N = 2$. Thus, $\{S_{t,k}\}_{k=1}^{K}$ is a 2-state Markov process with initial distribution $\delta^t(g_t) = (\delta_1^t(g_t), \delta_2^t(g_t))$ and t.p.m.

$$\Gamma^t(g_t) = \begin{bmatrix} \gamma_{11}^t(g_t) & \gamma_{12}^t(g_t) \\ \gamma_{21}^t(g_t) & \gamma_{22}^t(g_t) \end{bmatrix} = \begin{bmatrix} p & 1-p \\ 1-q & q \end{bmatrix}.$$

Let's define $F_t = \sum_{k=1}^{K} I(S_{t,k} = 2)$. We will denote the probability of the $i-th$ occurrence of being in state 2 at time k as $P(V^i = k)$. Therefore, for $i > 0$,

$$P(F_t = i) = \sum_{k=1}^{K-1} P(V^i = k) P(S_{t,j} = 1, k < j \leq K \mid S_{t,k} = 2) + P(V^i = K).$$

Notice that $P(F_t = 0) = P(S_{t,k} = 1 \text{ for all } k \in \{1, \ldots K\}) = \delta_1^t(g_t) p^{K-1}$.

How do we compute $P(V^i = k)$?

Waiting time distribution patterns for HMMs

Notice that $\{V \leq k\}$ (having entered state 2 by time k) can be viewed as the pattern $\Lambda(1) = 2$ having occurred by time k.

In general: given an unobserved state sequence $S_{t,1}, \ldots, S_{t,K}$ and a sequence of symbols of the set $\{1, 2\}$, or pattern, Λ, how can we compute the probability that Λ has occurred in the hidden state sequence by time $k \leq K$?

Key: Auxiliary Markov chains!

Waiting time distribution patterns for HMMs

Given $\{S_{t,k}\}$ and a pattern Λ, it is possible to construct a Markov chain $\{Z_k\}$ such that there's a one-to-one correspondence between classes of its states and those of $\{S_{t,k}\}$, with an absorbing state, Γ, defined to correspond to the occurrence of pattern of interest [Aston & Martin, 2007].

Such auxiliary Markov chain holds ([Aston & Martin, 2007]; Theorem 3.2)

$$P(Z_k = \Gamma) = \psi_1 \left(\prod_{j=2}^{k} M_j \right) U(\Gamma) \tag{3}$$

- M_j = t.p.m. for transitions from Z_{j-1} to Z_j
- ψ_1 = initial distribution of Z_1
- $U(\Gamma)$ = column vector with a one in the location corresponding to the absorbing state Γ, and zeroes elsewhere

Construction of states for $\{Z_k\}$

Let's consider the alternating run pattern of length exactly h, with h odd:

$$\Lambda(h) = \overbrace{2121\ldots212}^{h}.$$

Progress in the pattern can be

$$\{2, 21, 212, \ldots, \overbrace{2121\ldots21}^{h-1}\}.$$

Then, the states of S_Z are defined as ordered vectors pairs, the first element being the current value of the $\{S_{t,k}\}$ sequence, and the second element being the progress into a pattern:

$$(1, 21), (1, 2121), \ldots (1, \overbrace{212\ldots21}^{h-1}),$$

$$(2, 2), (2, 212), (2, 21212), \ldots (2, \overbrace{212\ldots12}^{h-2}), \Gamma.$$

The state $(1, \varnothing)$ is needed as a possible state for Z_1, where $\varnothing$ denotes that there's no progress in the pattern.

Computation of transition probabilities for $\{Z_k\}$

The transition probabilities are based on the transition probabilities for the $\{S_{t,k}\}$ sequence through the first entry of the ordered vector pairs, while the entire vector state from S_Z determines the possible destination of the state:

$$
\begin{aligned}
P(Z_k = (1, \overbrace{212\ldots21}^{i+1}) \mid Z_{k-1} = (2, \overbrace{212\ldots2}^{i})) \\
= P(S_{t,k} = 1 \mid S_{t,k-1} = 2), \quad i \text{ odd}, \; i = 1, \ldots, h-2
\end{aligned}
$$

$$
\begin{aligned}
P(Z_k = (2,2) \mid Z_{k-1} = (2, \overbrace{212\ldots2}^{i})) \\
= P(S_{t,k} = 2 \mid S_{t,k-1} = 2), \quad i \text{ odd}, \; i = 1, \ldots, h-2
\end{aligned}
$$

$$
\begin{aligned}
P(Z_k = \Gamma \mid Z_{k-1} = (1, \overbrace{212\ldots21}^{h-1})) \\
= P(S_{t,k} = 2 \mid S_{t,k-1} = 1)
\end{aligned}
$$

$$
P(Z_k = \Gamma \mid Z_{k-1} = \Gamma) = 1
$$

Special case: h = 1

For $h = 1$, $\Lambda(1) = 2$, and the states of S_Z are

$$(1, \varnothing), \Gamma,$$

and the transitions probabilities result in

$$P(Z_k = (1, \varnothing) \mid Z_{k-1} = (1, \varnothing)) \\ = P(S_{t,k} = 1 \mid S_{t,k-1} = 1) = p$$

$$P(Z_k = \Gamma \mid Z_{k-1} = (1, \varnothing)) = \\ P(S_{t,k} = 2 \mid S_{t,k-1} = 1) = 1 - p$$

$$P(Z_k = \Gamma \mid Z_{k-1} = \Gamma) = 1$$

Therefore, for each $j > 1$, $M_j = M_z$, where $M_z = \begin{bmatrix} p & 1-p \\ 0 & 1 \end{bmatrix}$, and $\psi_1 = \delta^t(g_t)$.

Equation (3) can be simplified as

$$P(Z_k = \Gamma) = \psi_1 \left(\prod_{j=2}^{k} M_j \right) U(\Gamma) = \delta^t(g_t) M_z^{k-1} U(\Gamma), \quad U(\Gamma) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{4}$$

Initial goal:

$$P\left(F_t = i\right) = \sum_{k=1}^{K-1} P(V^i = k) P(S_{t,j} = 1, k < j \leq K \mid S_{t,k} = 2) + P(V^i = K).$$

From (4), it follows for $k > 1$,

$$P(V = k) = P(V \leq k) - P(V \leq k-1) = \delta \left[M_z^{k-1} - M_z^{k-2} \right] U(\Gamma).$$

Denoting $M_1 = I$, $M_2 = M_z - I$ and $M_k = M_z M_{k-1}$ for $k \geq 3$, we have

$$P(V = k) = \delta_1^t(g_t) M_k U(\Gamma).$$

Furthermore, for occurrence $i > 1$,

$$
\begin{aligned}
P(V^i = k) &= \sum_{k_1=1}^{k-1} P(V^i = k \mid V^{i-1} = k_1) P(V^{i-1} = k_1) \\
&= \sum_{k_1=1}^{k-1} P(V^{i-1} = k_1) P_\eta(V = k - k_1)
\end{aligned}
$$

where $P_\eta(V = k) = (1-q) M_k U(\Gamma)$.

$$
P\left(F_t = i\right) = \sum_{k=1}^{K-1} P(V^i = k) {\color{red} P(S_{t,j} = 1, k < j \leq K \mid S_{t,k} = 2)} + P(V^i = K).
$$

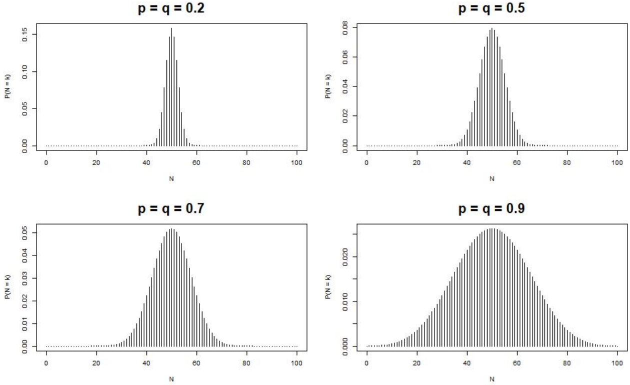
We have that

$$
\begin{aligned}
P(S_{t,j} = 1, k < j \leq K \mid S_{t,k} = 2) &= P(S_{t,k+1} = 1 \mid S_{t,k} = 2) \prod_{j=k+1}^{K} \big(\Gamma^t(g_t)\big)_{11} \\
&= (1-q) p^{K-(k+1)}.
\end{aligned}
$$

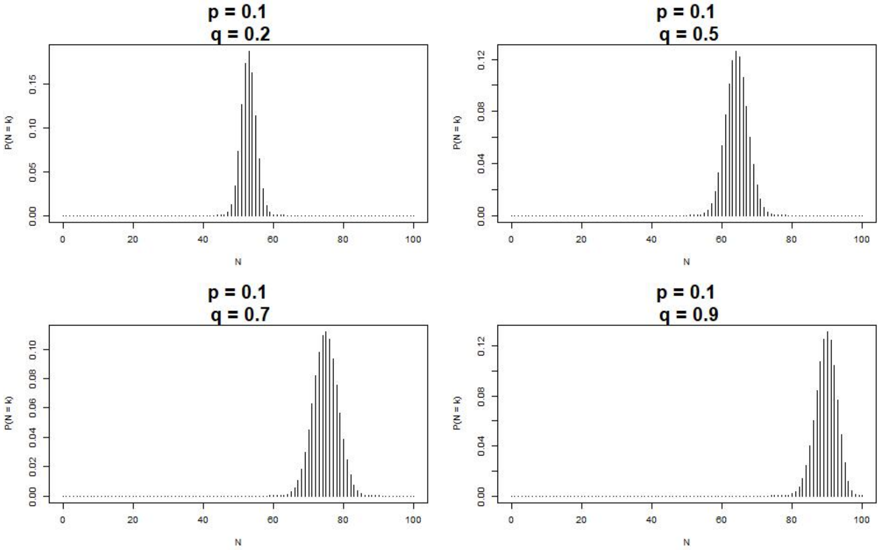
Simulation of $F_t = \sum_{k=1}^{K} \mathrm{I}(S_{t,k} = \text{foraging})$

Let's recall the t.p.m. of $\{S_{t,k}\}_{k=1}^{K}$ is given by $\Gamma^t(g_t) = \begin{bmatrix} p & 1-p \\ 1-q & q \end{bmatrix}$.

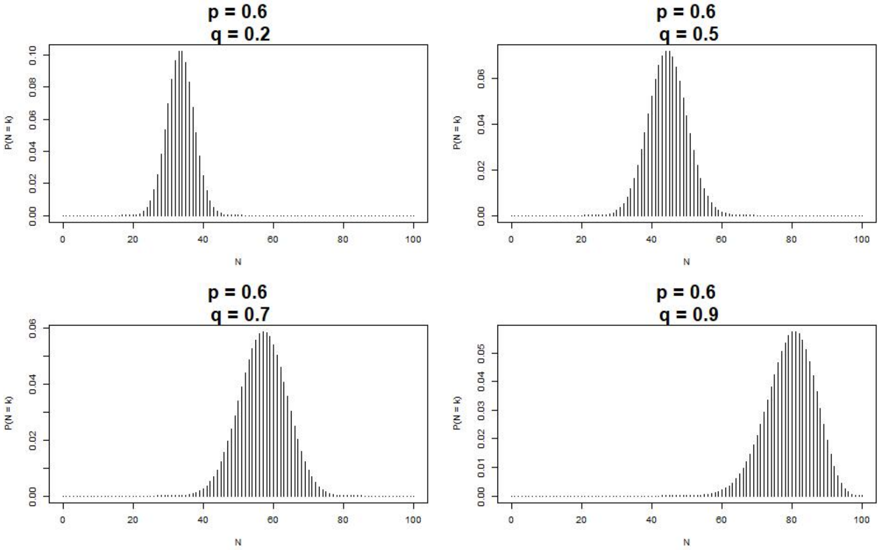
Considering $K = 100$, and taking $\delta^t(g_t) = (1/2, 1/2)$, the distribution of F_t was computed for different values of p and q.



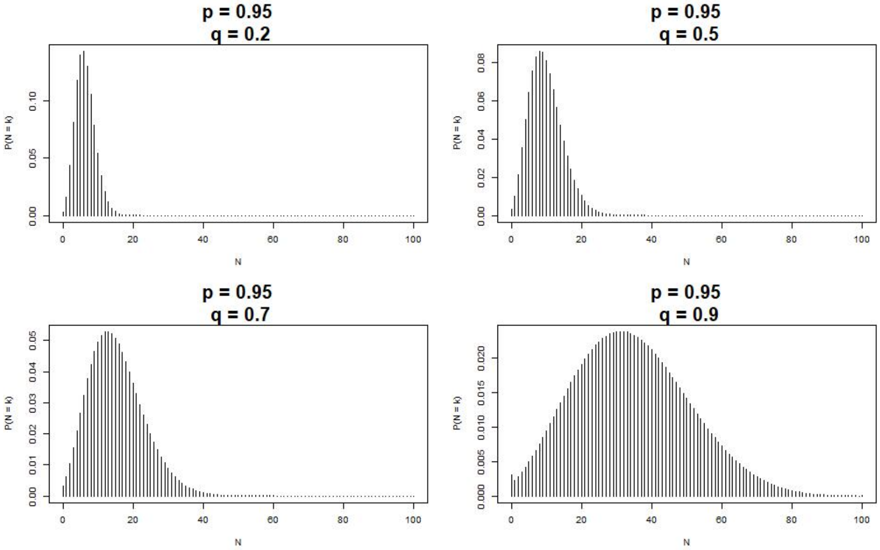
Fixing $p = 0.1$ and varying q



Fixing $p = 0.6$ and varying q

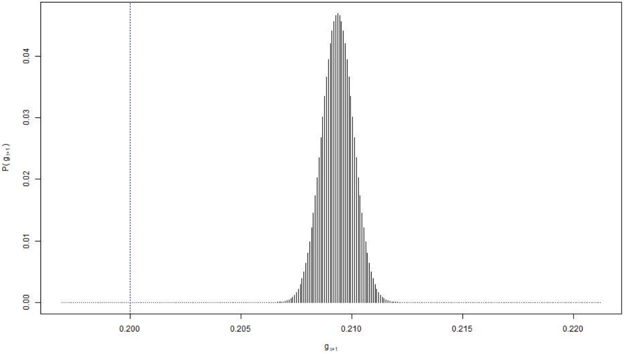


Fixing $p = 0.95$ and varying q



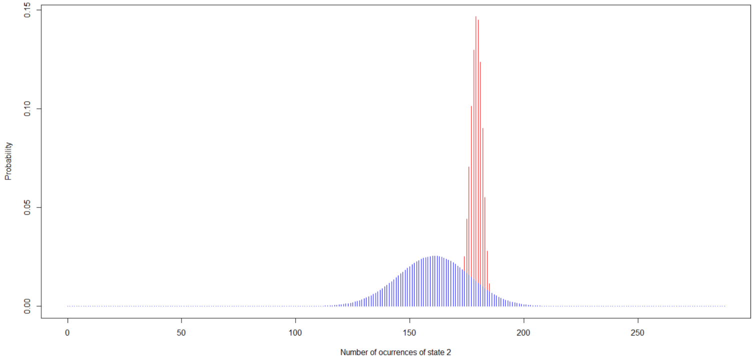
Prediction of animal condition (g_t)

For $K = 288$ (1 day), $w_t = 70$kg, $g_t = 0.2$ (which implies $b_t = 0.2w_t = 14$kg), $\sum_{k=1}^{K} d_{t,k} = 1.65$km and $p = q = 0.5$, the following prediction distribution results:



Distribution of F_t before and after incorporating observed process

Number of ocurrences of state $2 = 179$ (out of $K = 288$)



Summary

- General framework to model both positional and physiological data, with percentage body fat used as a proxy for sheep's overall body condition
- Incorporation proposal of animal behaviour dynamics along with biological equations, E_t, for the prediction of latent processes b_t and m_t (related to g_t)
- Computation of the distribution of F_t, in which is encapsulated the animal behaviour information
- Prediction of g_{t+1}, assuming w_t, b_t is observed

Future work

- Extend the model to more states $(N \geq 3)$
- Setting structure to incorporate b_t, m_t
- Model implementation in Stan

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Thank you!