Mixed HMMs: *a promising tool for model selection*

Fanny Dupont, Marianne Marcoux, Nigel Hussey, Marie Auger-Méthé

It is crucial to understand narwhals ’ normal behavior

Decadal migration phenology of a long-lived Arctic icon keeps pace with climate change

Courtney R. Shuert, Marianne Marcoux, Nigel E. Hussey, +2, and Marie Auger-Méthé Authors Info & Affiliations

Tracking arctic marine mammal resilience in an era of rapid ecosystem alteration

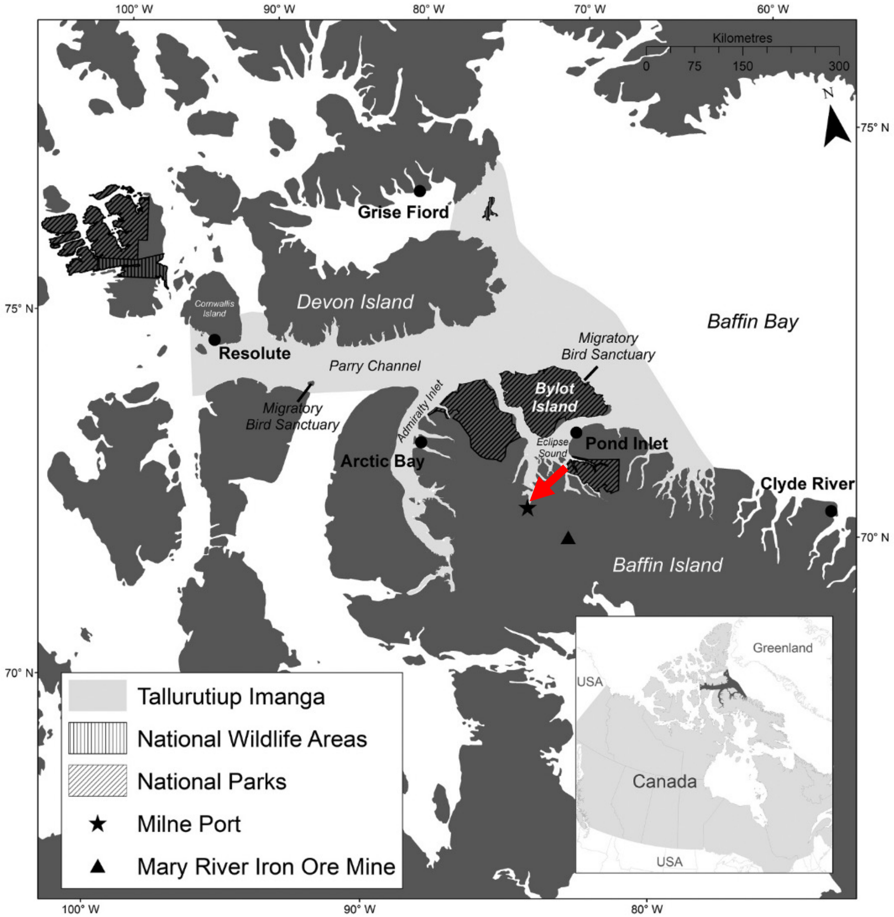
Sue E. Moore, Randall R. Reeves

Published: October 9, 2018 • https://doi.org/10.1371/journal.pbio.2006708

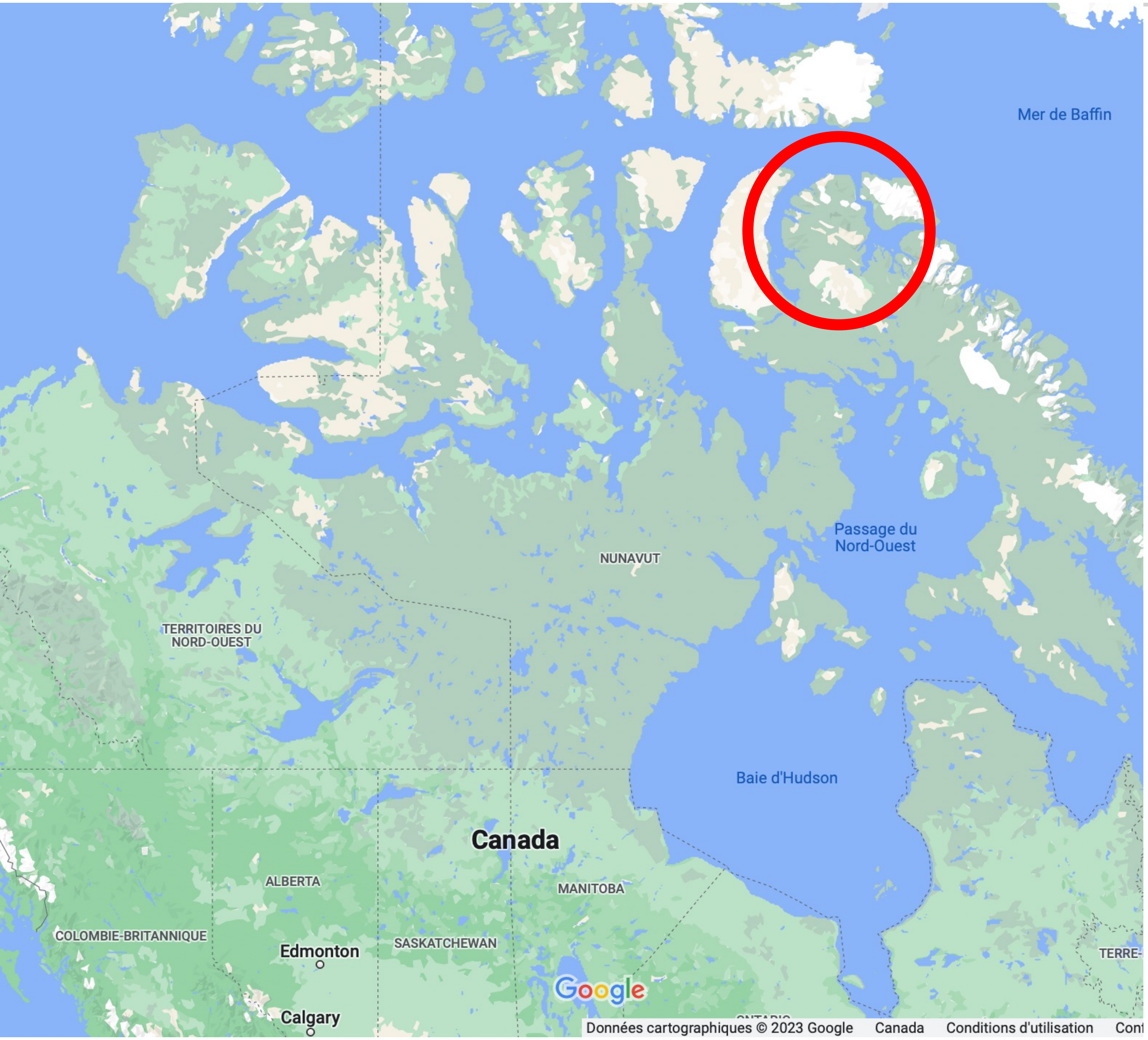
Evidence suggests potential transformation of the Pacific Arctic ecosystem is underway

Henry P. Huntington, Seth L. Danielson, Francis K. Wiese, Matthew Baker, Peter Boveng, John J. Citta, Alex De Robertis, Danielle M. S. Dickson, Ed Farley, J. Craighead George, Katrin Iken, David G. Kimmel, Kathy Kuletz, Carol Ladd, Robert Levine, Lori Quakenbush, Phyllis Stabeno, Kathleen M. Stafford, Dean Stockwell & Chris Wilson

Nature Climate Change **10**, 342–348 (2020) | Cite this article



Narwhal location data



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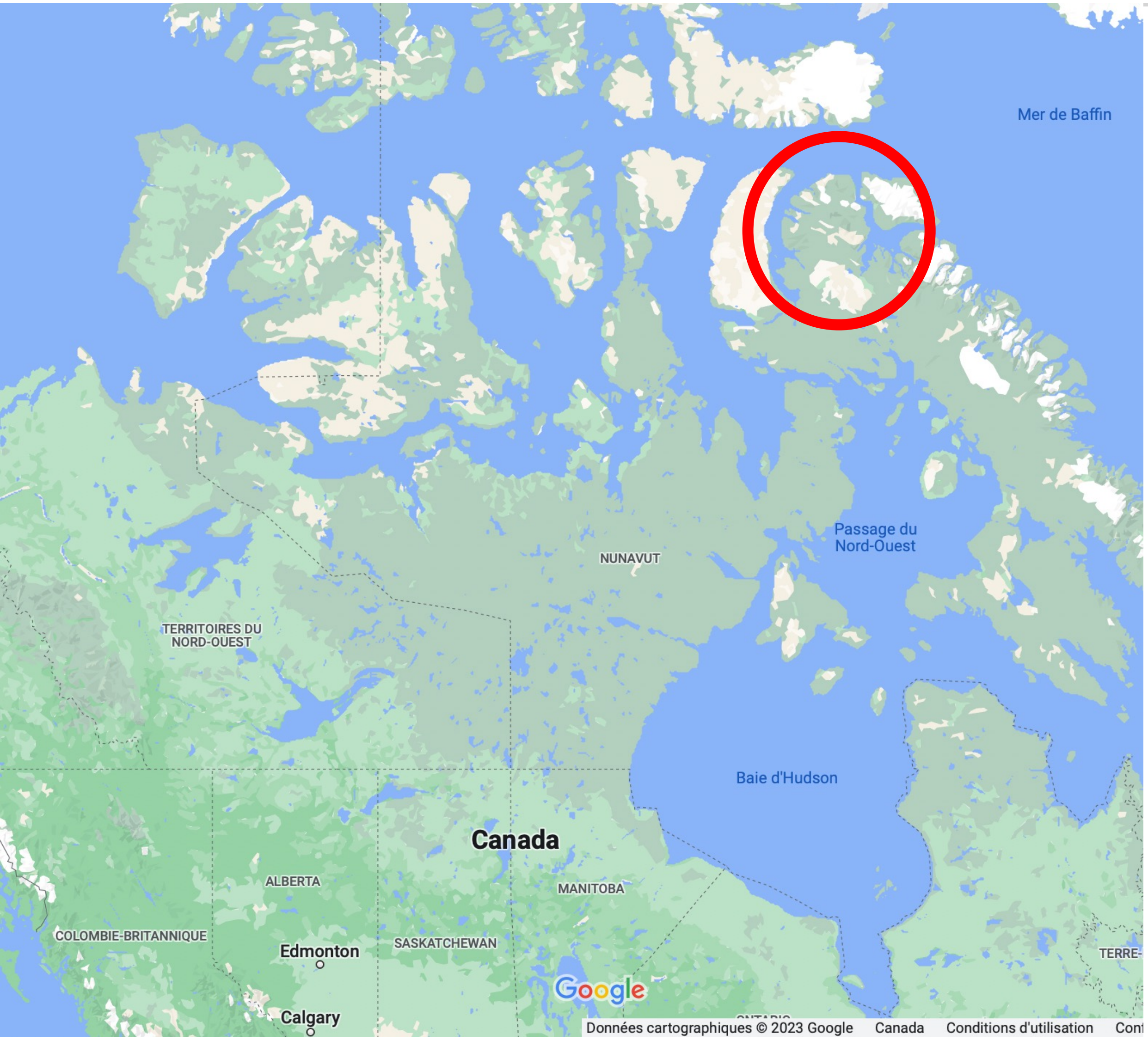
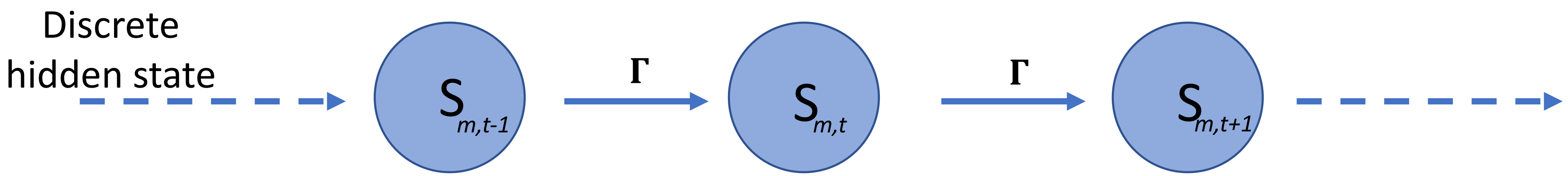


- Tremblay Sound, Nunavut, Canada.
- One colour per individual
- 8 individuals.
- One month data, resolution of one hour.

HMMs are an important modeling framework for analyzing ecological time series.



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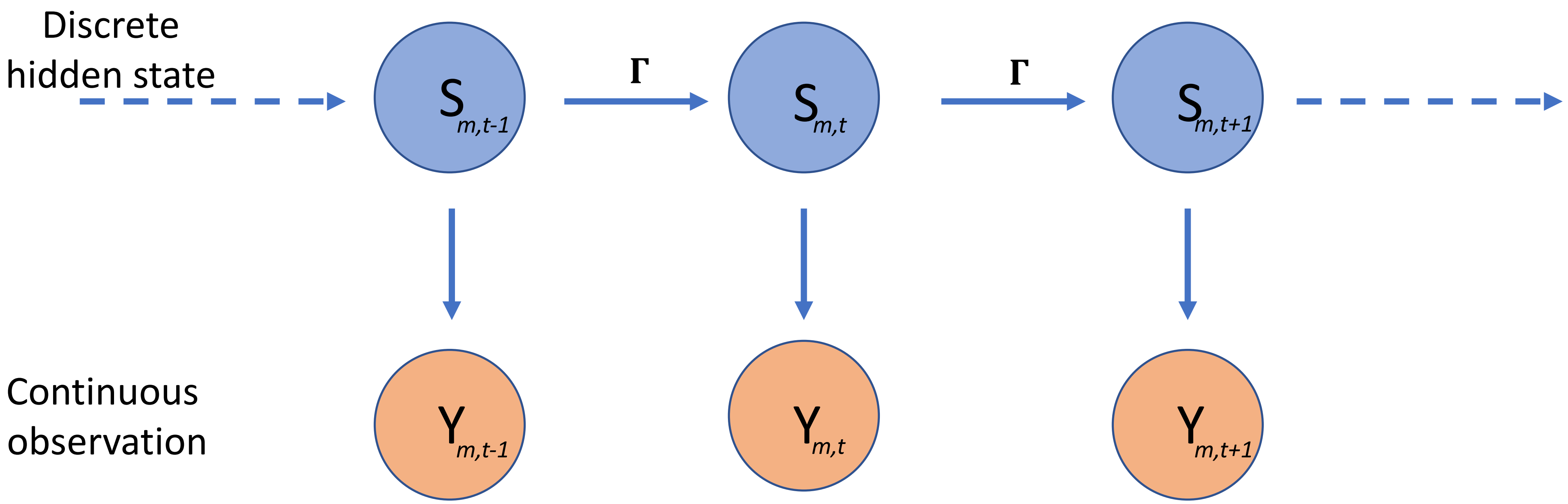

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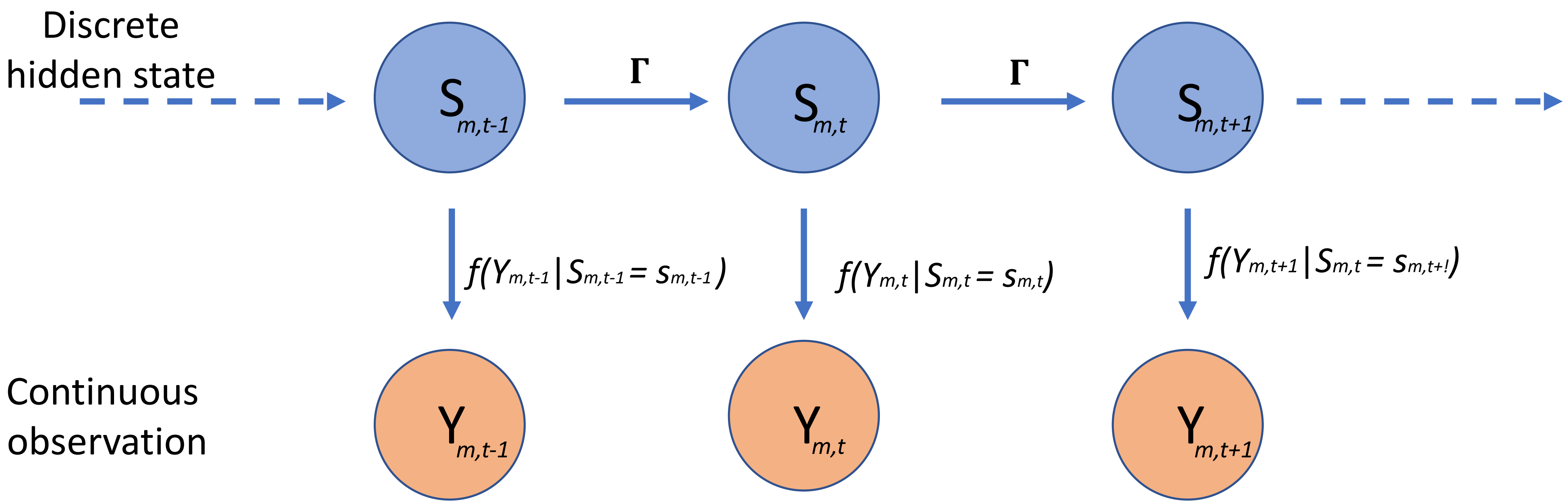
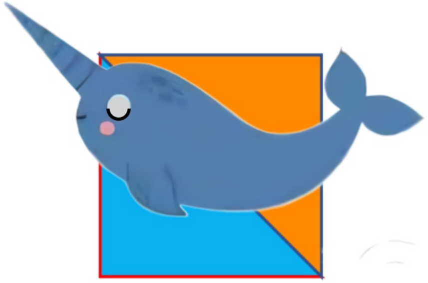


EXAMPLE: the hidden state of the narwhal defines the step length observed



t-1

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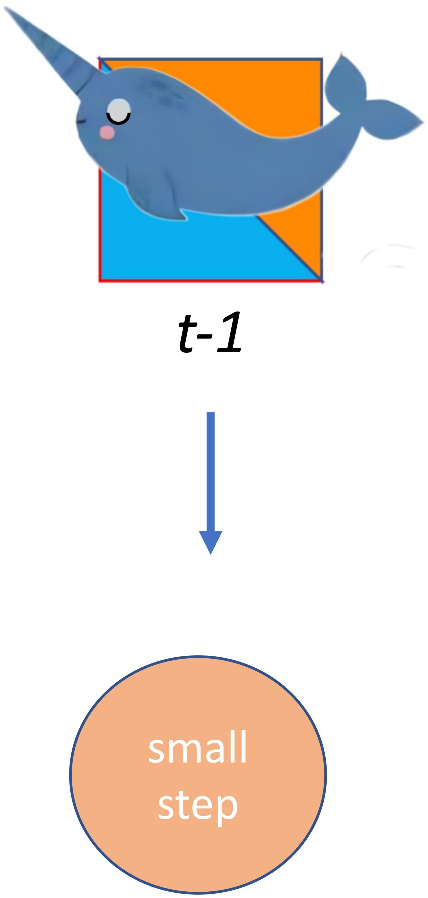

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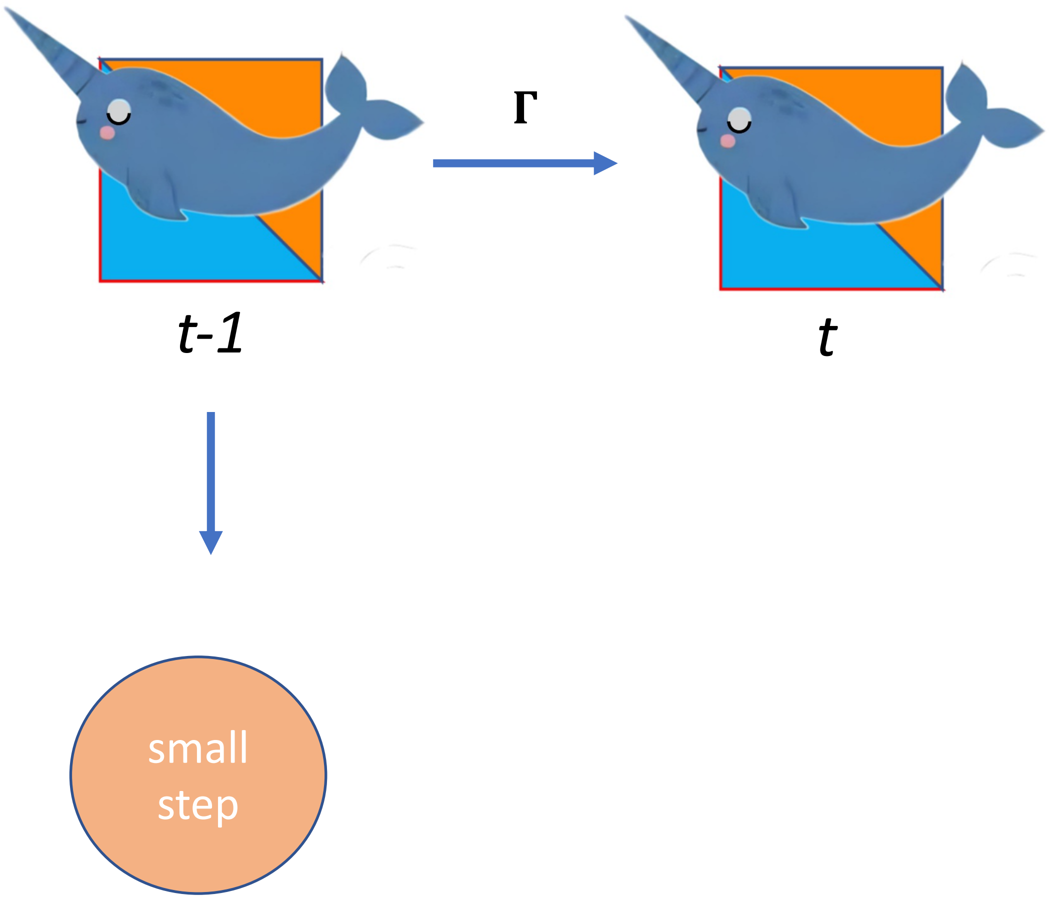

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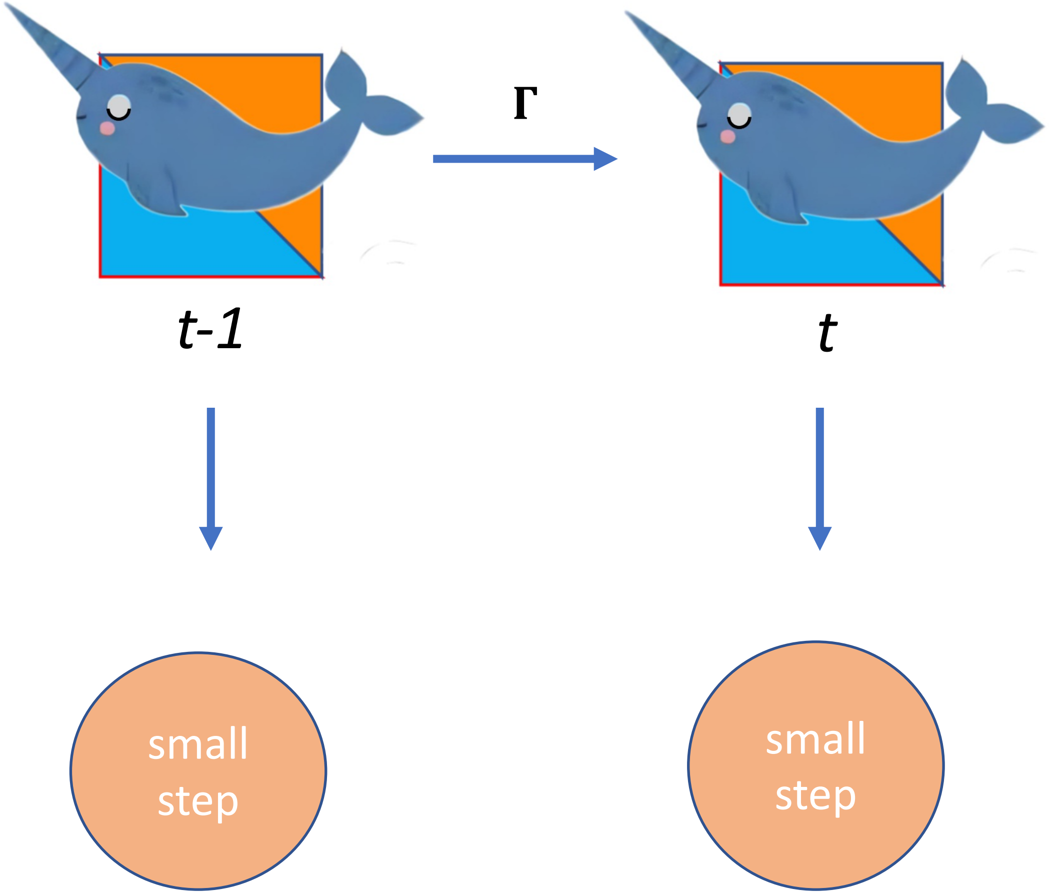

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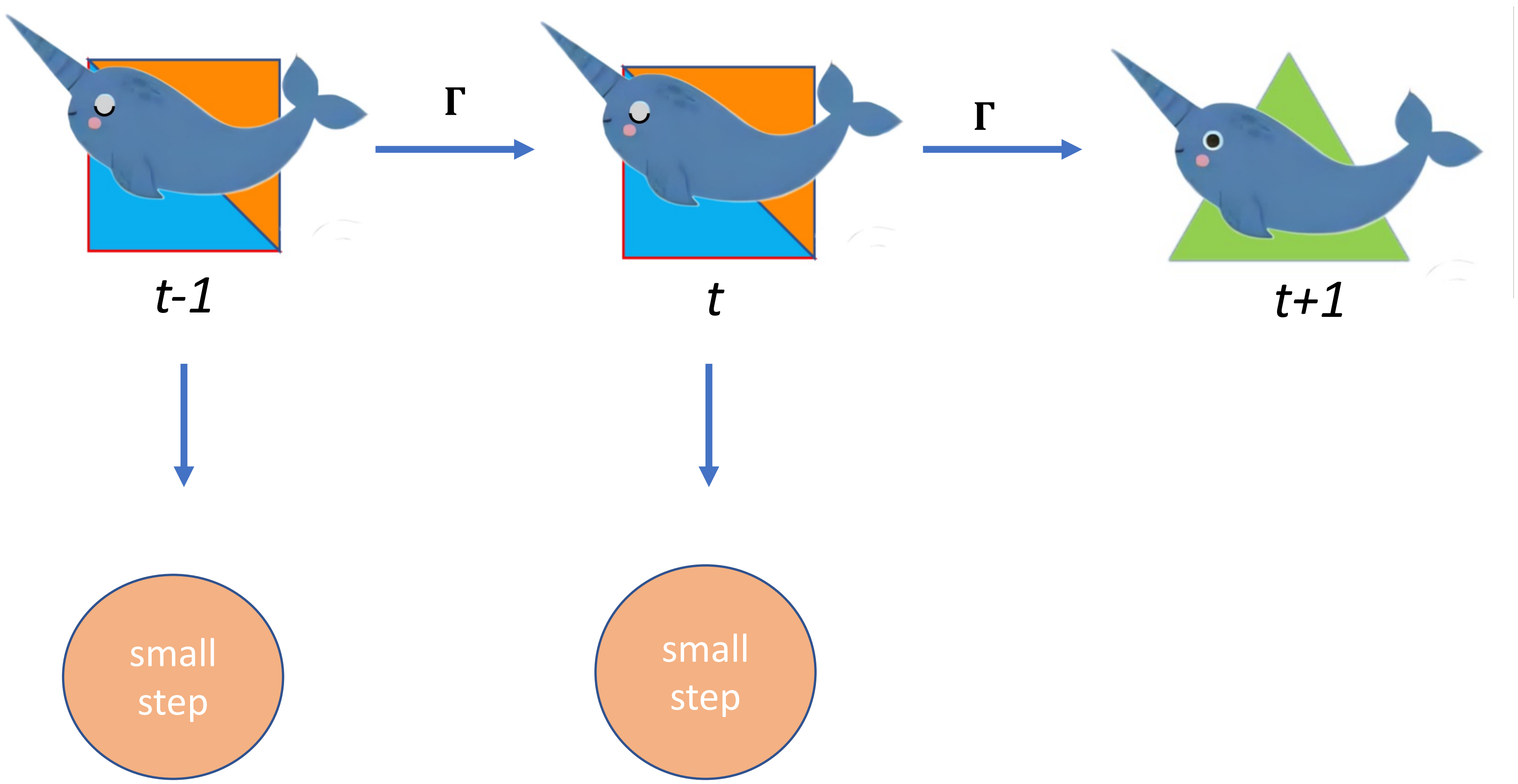

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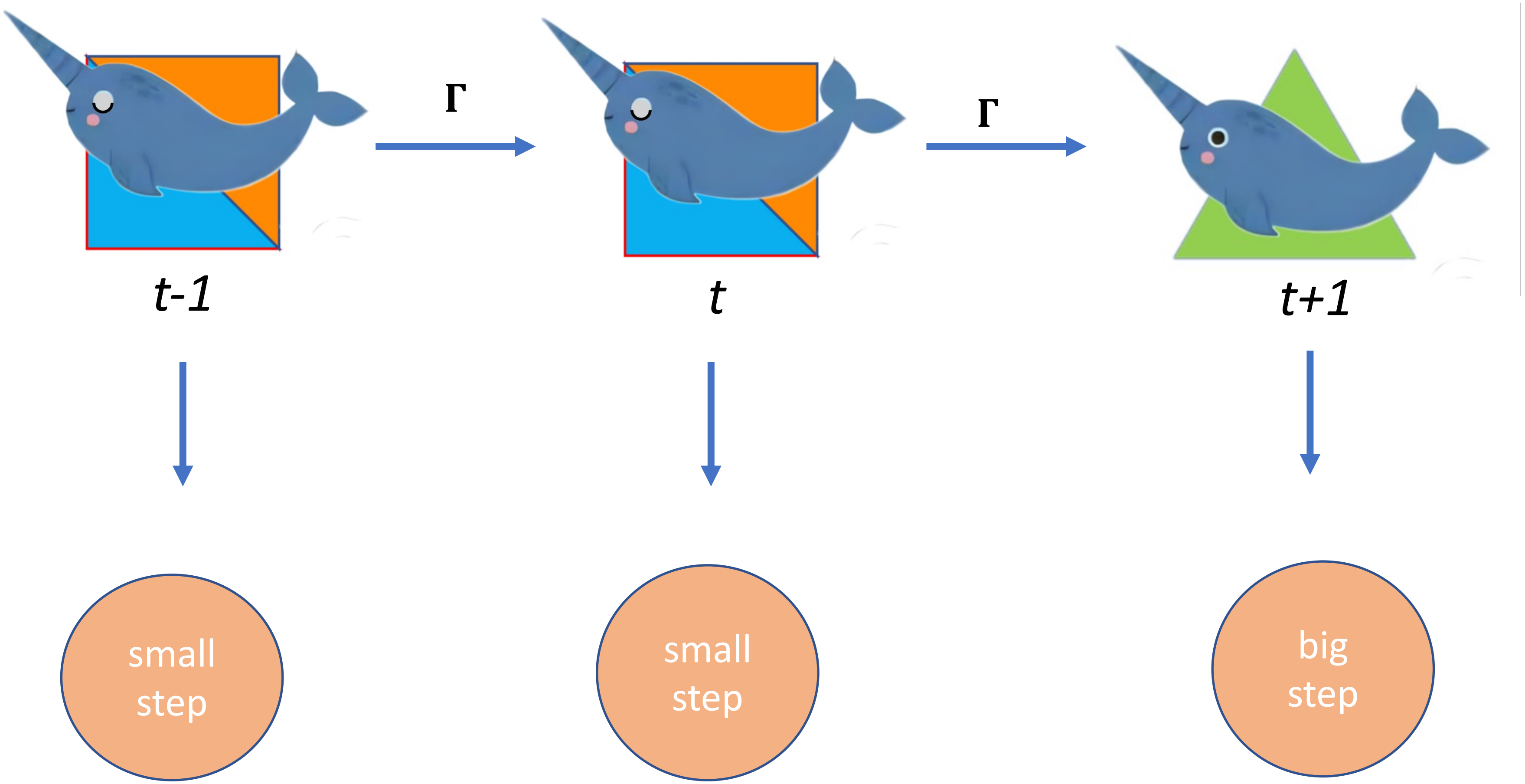

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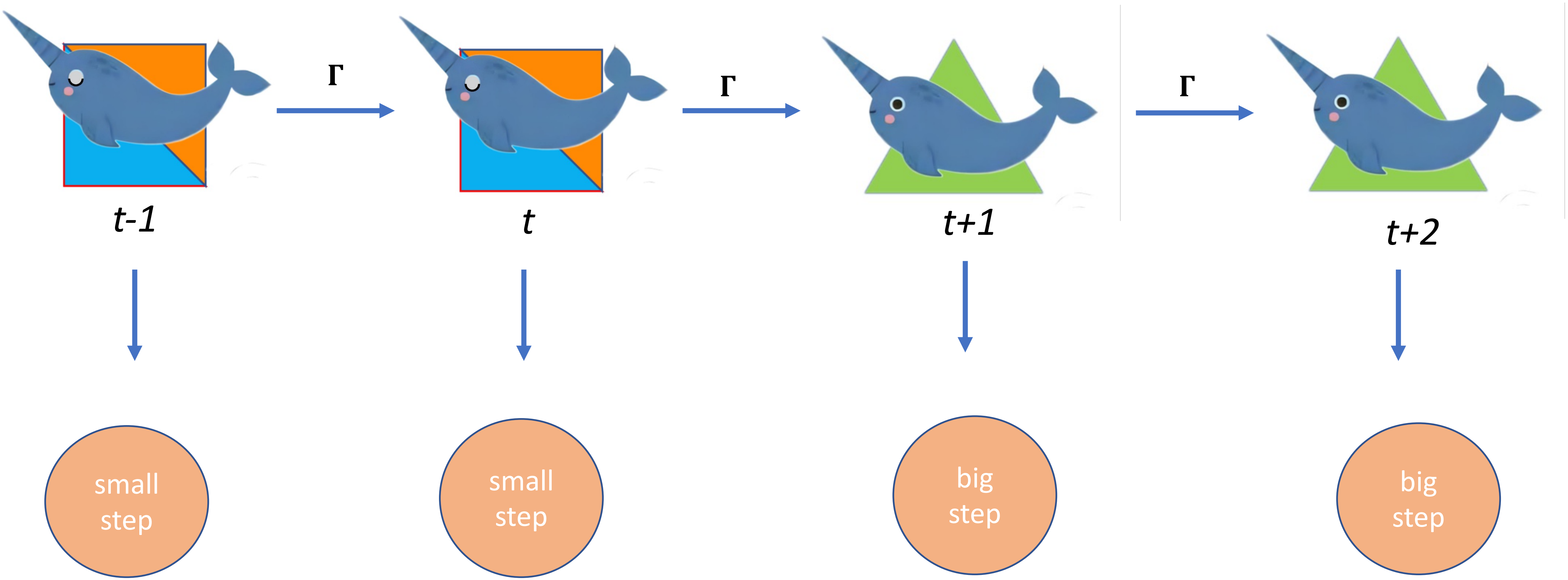
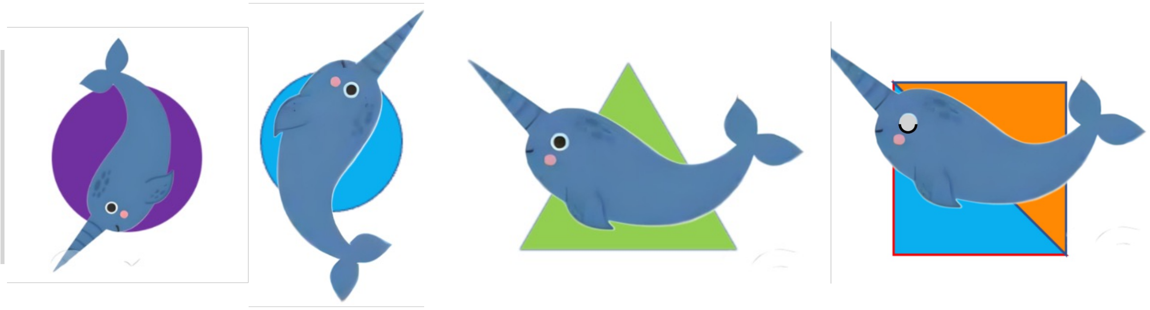


It is hard to determine the number of states



- Prior knowledge about the animal's behavior

- Select the number of states with information criteria (AIC, BIC)

Case Study

Narwhal case study

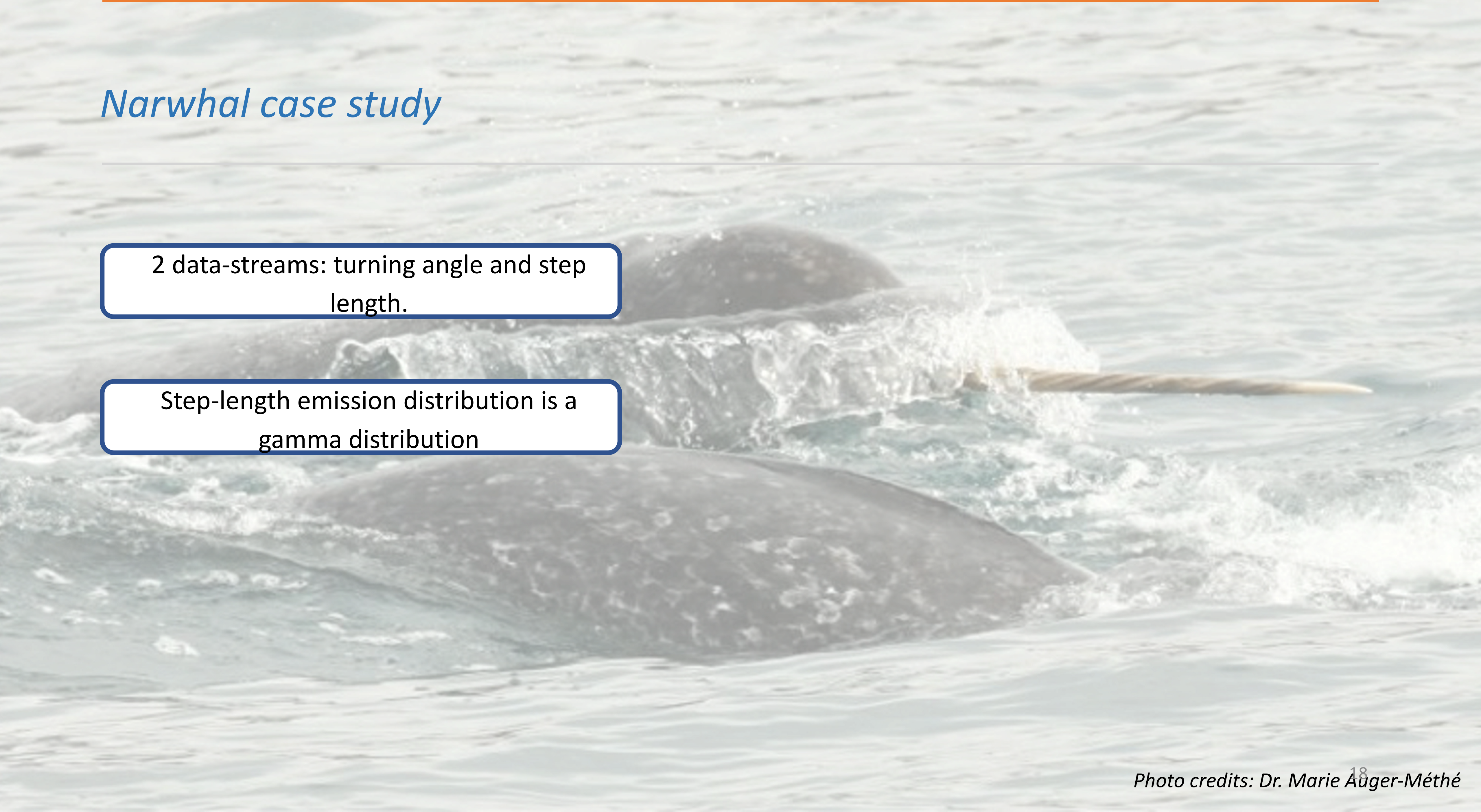
2 data-streams: turning angle and step length.

Photo credits: Dr. Marie Auger-Méthé

Narwhal case study

2 data-streams: turning angle and step length.

Step-length emission distribution is a gamma distribution



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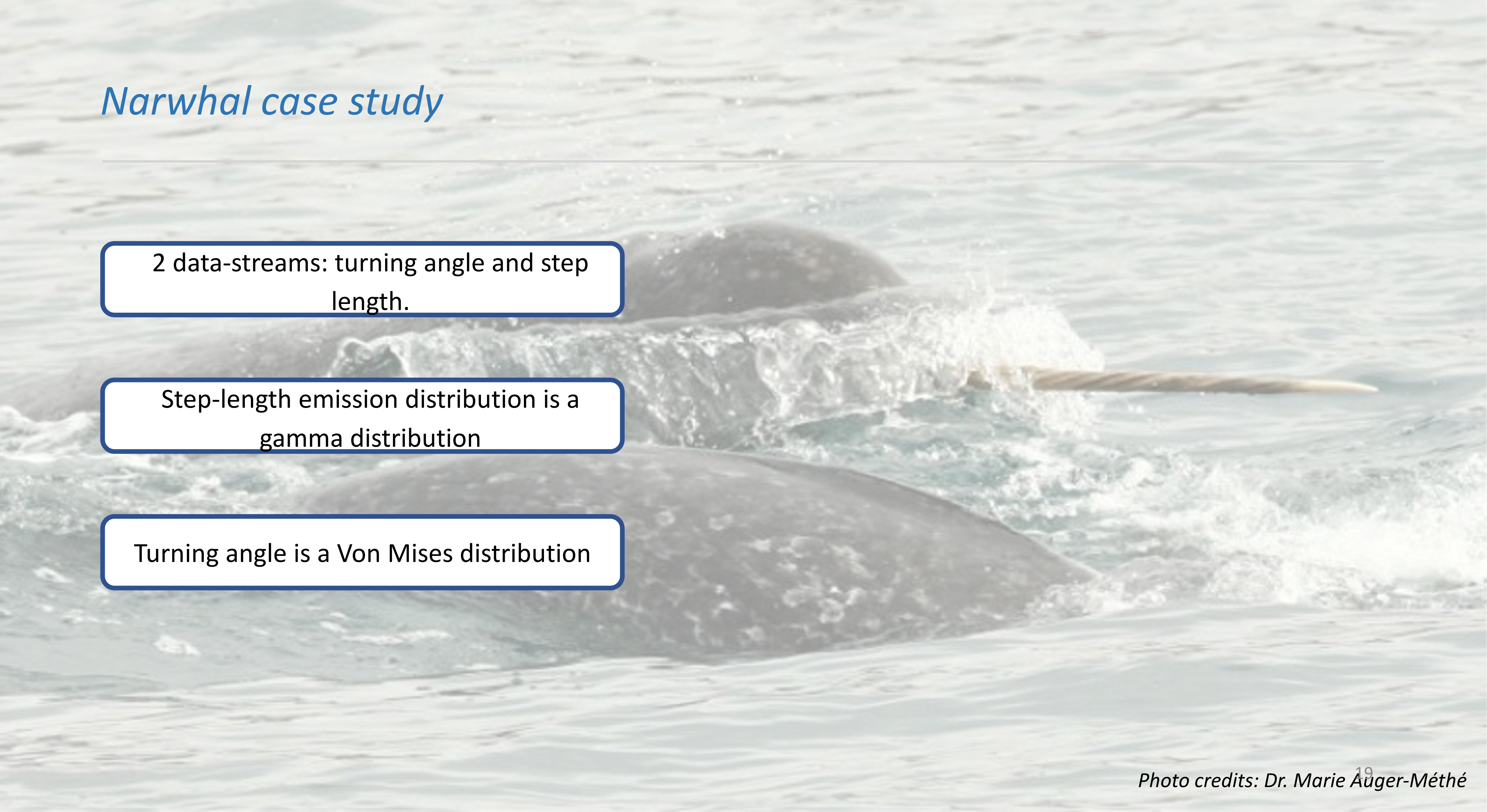

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Photo credits: Dr. Marie Auger-Méthé

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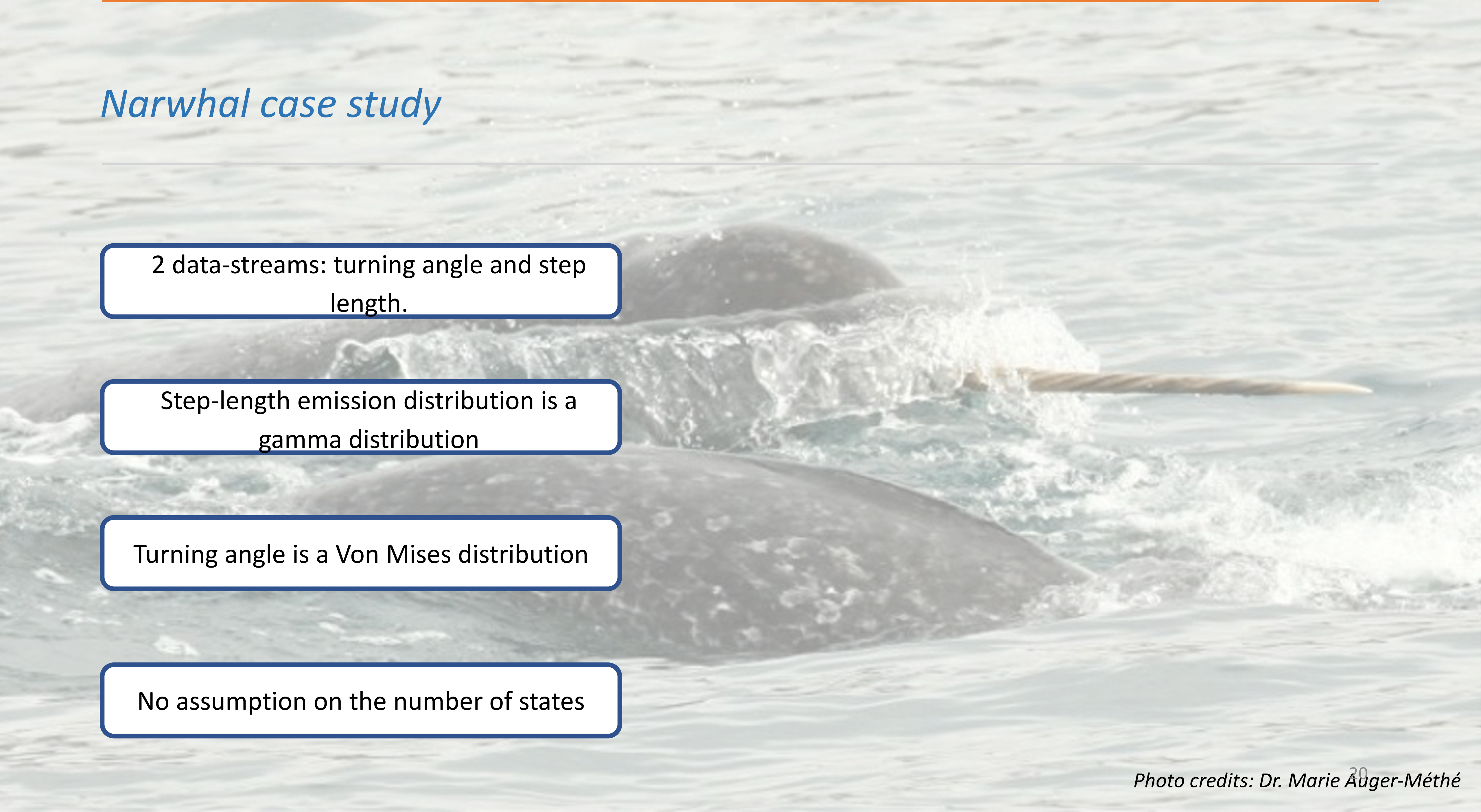

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No assumption on the number of states

Photo credits: Dr. Marie Auger-Méthé

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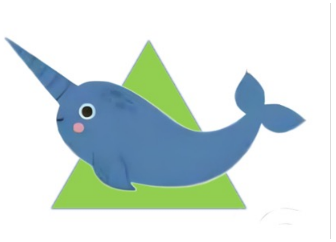
Turning angle is a Von Mises distribution

No assumption on the number of states

Select the best-supported model for modelling narwhals' behaviour.

Photo credits: Dr. Marie Auger-Méthé

Standard HMM selected 4 states



Travelling

Pohle, Jennifer, et al. "Selecting the number of states in hidden Markov models: pragmatic solutions illustrated using animal movement." *Journal of Agricultural, Biological and Environmental Statistics* 22 (2017): 270-293.

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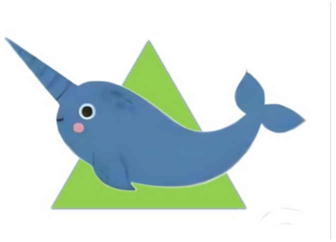

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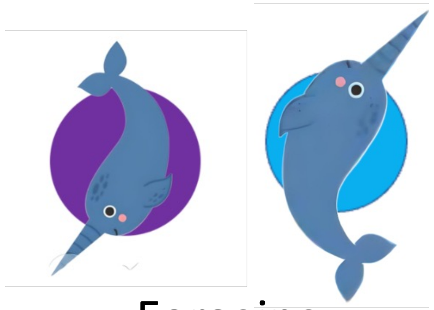

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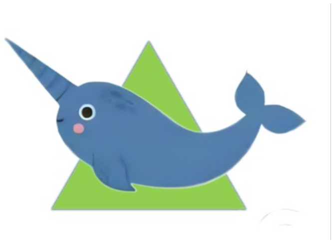

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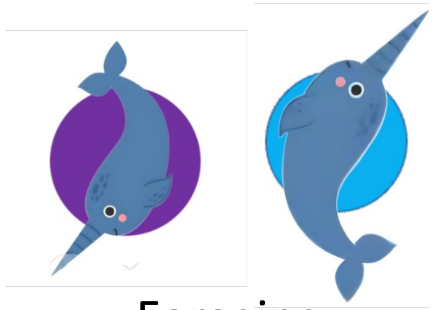

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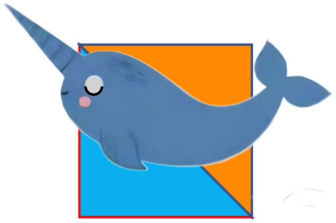

Sleeping

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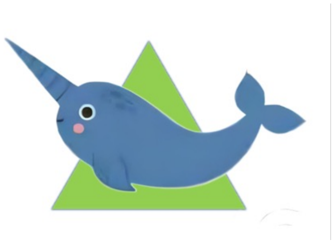

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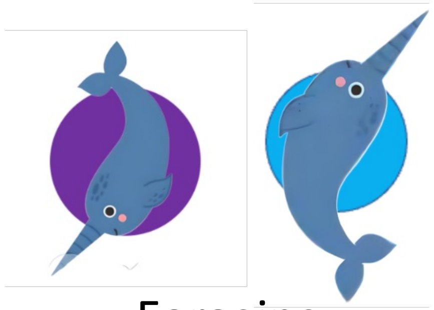

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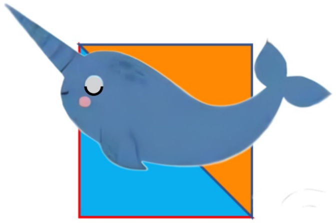

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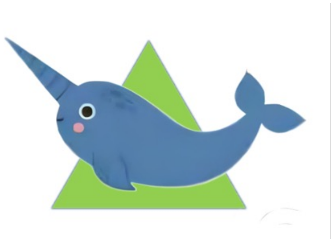

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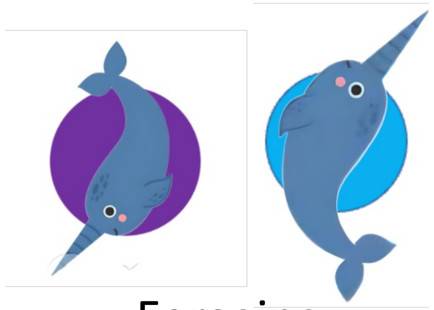

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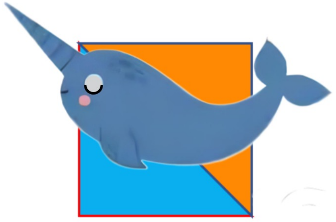

Sleeping

- Information criteria tend to favor models with numbers of states that are undesirably large under misspecification.

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Mixed HMMs

Random effects in HMMs

Mixed HMMs are a promising tool to select the right number of states

- **Four** different mixed models: derived from the common standard HMM.

Mixed HMMs are a promising tool to select the right number of states

The common standard HMM

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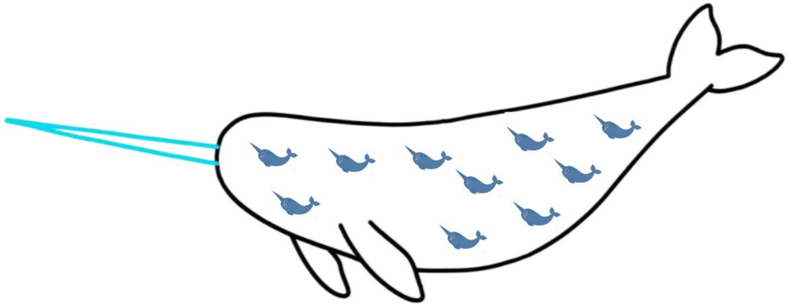
The common standard HMM

- No individual effects on parameters,

- Independence between individuals



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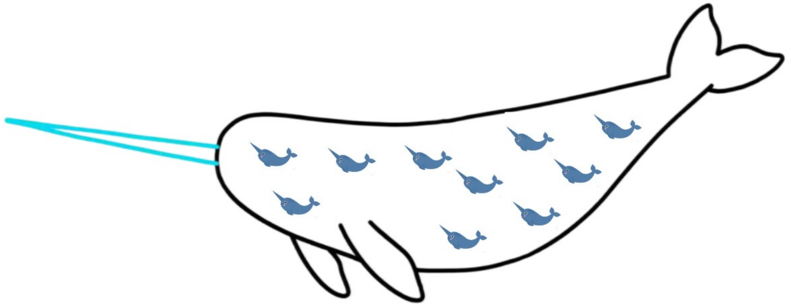

$$\mathscr{L}_{std} = \prod_{m=1}^{M} \delta \Gamma \mathbf{P}\left(\mathbf{y}_{m,1}\right) \Gamma \mathbf{P}\left(\mathbf{y}_{m,2}\right) \cdots \Gamma \mathbf{P}\left(\mathbf{y}_{m,T_m-1}\right) \Gamma \mathbf{P}\left(\mathbf{y}_{m,T_m}\right) \mathbf{1}, \quad (1)$$

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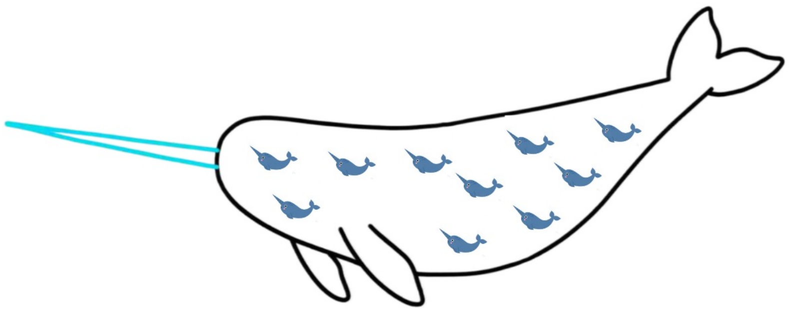

Number of individuals

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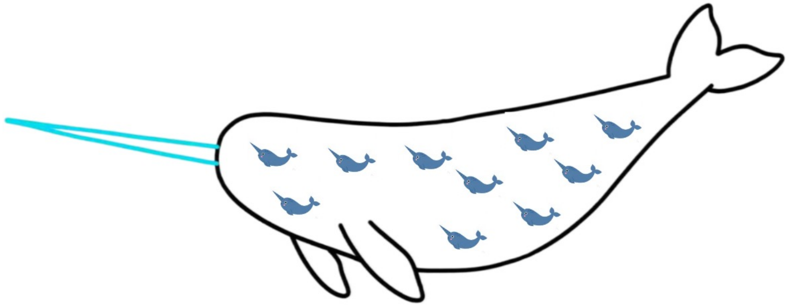

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Initial state probabilities vector

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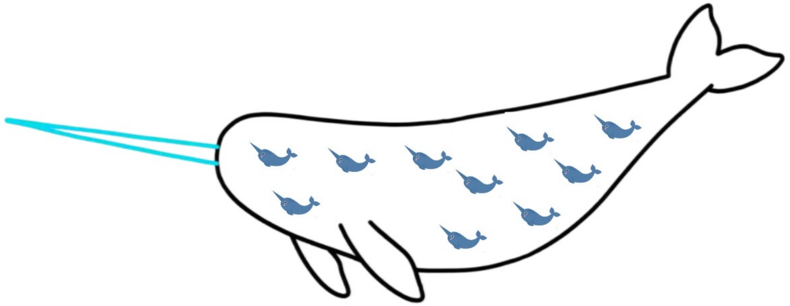

Number of individuals

conditional probability density of the observation of the m th individual given the hidden state at time 1.

$$\mathcal{L}_{std} = \prod_{m=1}^{M} \delta \Gamma \mathbf{P}\left(\mathbf{y}_{m,1}\right) \Gamma \mathbf{P}\left(\mathbf{y}_{m,2}\right) \cdots \Gamma \mathbf{P}\left(\mathbf{y}_{m,T_m-1}\right) \Gamma \mathbf{P}\left(\mathbf{y}_{m,T_m}\right) \mathbf{1}, \quad (1)$$

Initial state probabilities vector

Null model: each individual has its own standard HMM

- Highly parameterized model.

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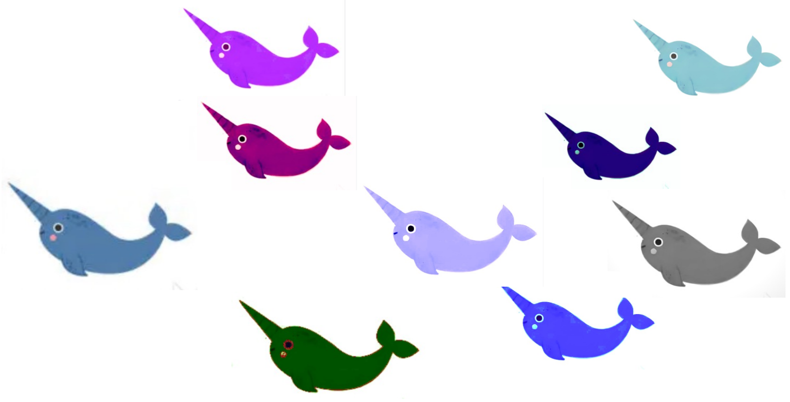

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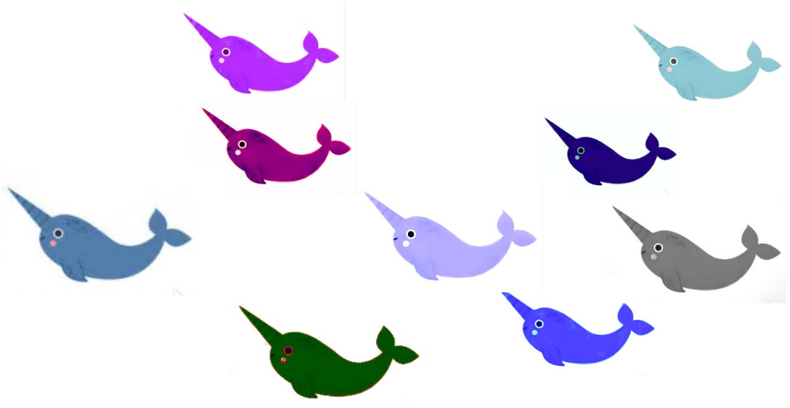

The complete likelihood for this model is:

$$\mathcal{L}_{null} = \prod_{m=1}^{M} \boldsymbol{\delta}_m \Gamma_m \mathbf{P}_m(\mathbf{y}_{m,1}) \Gamma_m \mathbf{P}_m(\mathbf{y}_{m,2}) \cdots \Gamma_m \mathbf{P}_m(\mathbf{y}_{m,T_m-1}) \Gamma_m \mathbf{P}_m(\mathbf{y}_{m,T_m}) \mathbf{1}, \quad (2)$$

Discrete-valued random effects: based on finite mixtures

- K components, K is chosen a priori using model selection criteria.
- A transition matrix per component.

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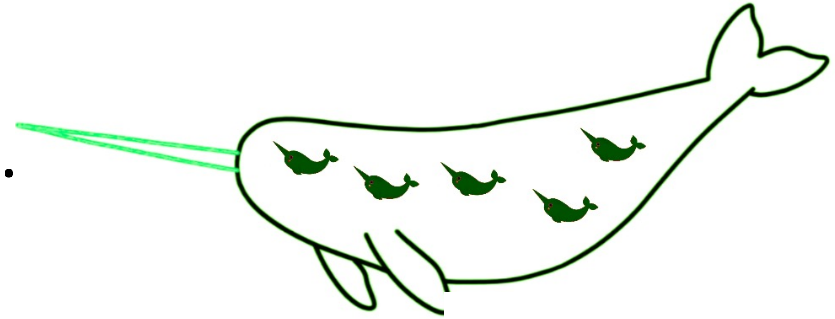

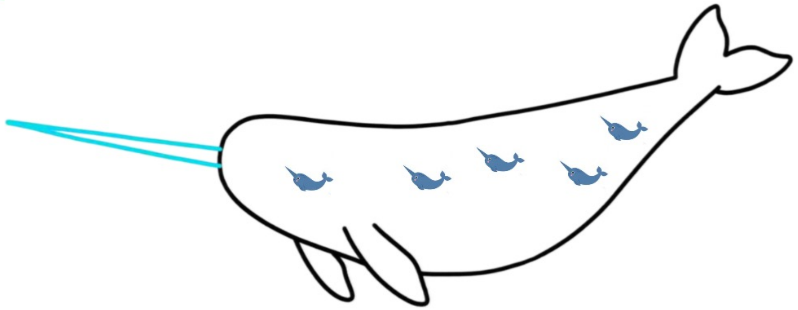

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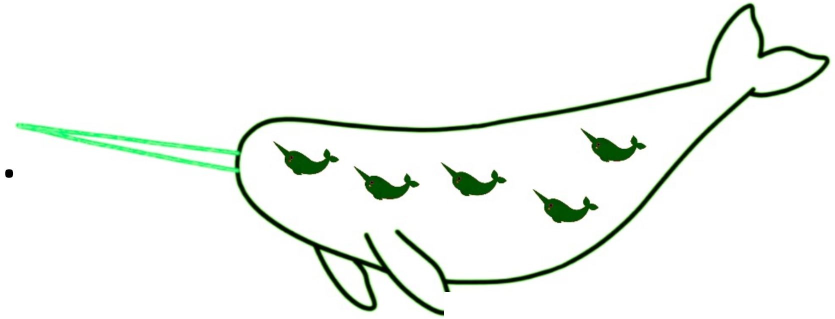

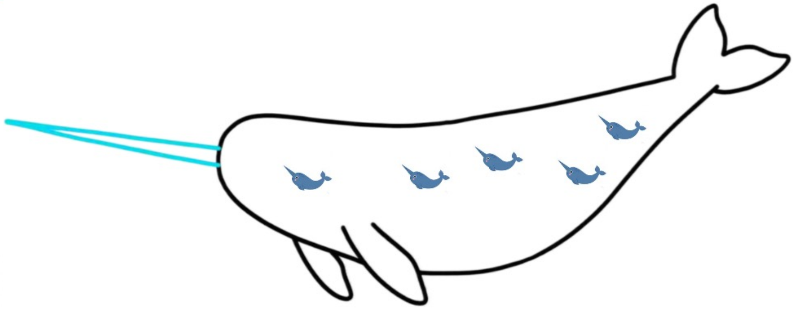

The complete likelihood for this model is:

$$\mathcal{L}_{\text{mix}} = \prod_{m=1}^{M} \sum_{k=1}^{K} \delta^{(k)} \Gamma^{(k)} \mathbf{P}\left(\mathbf{y}_{m,1}\right) \Gamma^{(k)} \mathbf{P}\left(\mathbf{y}_{m,2}\right) \cdots \Gamma^{(k)} \mathbf{P}\left(\mathbf{y}_{m,T_m-1}\right) \Gamma^{(k)} \mathbf{P}\left(\mathbf{y}_{m,T_m}\right) \mathbf{1} \pi^{(k)},$$

Continuous random effects: normal distribution

- Random effect $z_{m,i,j} \sim N(\mu_{i,j}, \sigma^2_{i,j})$ for $i \neq j$, $z_{m,i,j} = 0$ for $i = j$.

- Transition probability for the m-th individual $\gamma_{m,i,j} = \dfrac{\exp(z_{m,i,j})}{\sum_{l=1}^{N} \exp(z_{m,i,l})}$

Such that complete likelihood for this model is:

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Such that complete likelihood for this model is:

$$\mathcal{L}_{\text{cont}} = \prod_{m=1}^{M} \int_{\mathcal{Z}} \delta_m \Gamma_m \mathbf{P}\left(\mathbf{y}_{m,1}\right) \Gamma_m \mathbf{P}\left(\mathbf{y}_{m,2}\right) \cdots \Gamma_m \mathbf{P}\left(\mathbf{y}_{m,T_m-1}\right) \Gamma_m \mathbf{P}\left(\mathbf{y}_{m,T_m}\right) 1 f\left(\mathbf{z}_m | \mu, \sigma\right) d\mathbf{z}_m, \tag{4}$$

where $f\left(\mathbf{z}_m | \mu, \sigma\right) = \prod_{i=1}^{N} \prod_{j \neq i} f\left(z_{m,i,j} | \mu_{i,j}, \sigma_{i,j}\right)$ is the joint density of $\mathbf{z}_m = \left(z_{m,i,j}\right)_{i \neq j}$

Continuous random effects: t distribution

- Random effect $z_{m,i,j} \sim t_{df}$ for $i \neq j$, $z_{m,i,j} = 0$ for $i = j$.
- df is the degree of freedom
- Fatter tail than normal distribution

Simulation framework

Mixed HMMs are a promising tool to select the right number of states

Simulation framework

Simulate 10 individuals under a scenario
(two-state gamma HMM)

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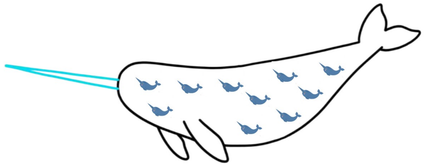
Standard HMM



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Simulate 10 individuals under a scenario
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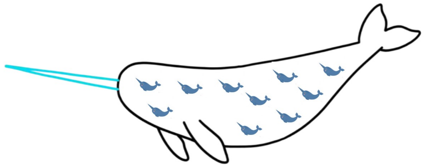
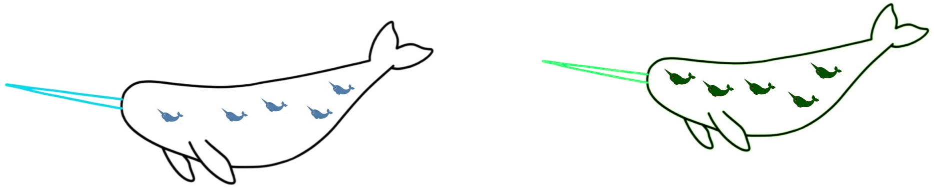


Mixture with two components



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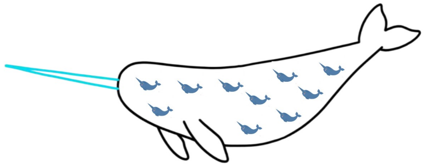

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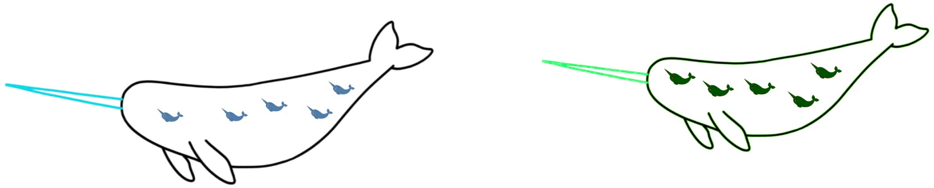
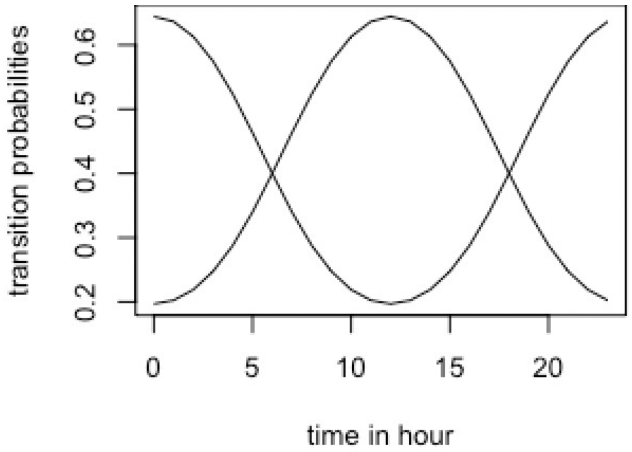


Temporal Variation



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Simulate 10 individuals under a scenario (two-state gamma HMM)

↓

Fit the four models with 2, 3, and 4 states

Mixed HMMs are a promising tool to select the right number of states

Simulation framework

Simulate 10 individuals under a scenario (two-state gamma HMM)

↓

Fit the four models with 2, 3, and 4 states

↓

Select the number of states, for each of the five models, with AIC and BIC.

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Analyze the proportion of states selected over 100 replications

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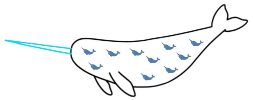
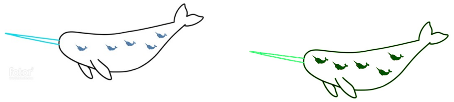
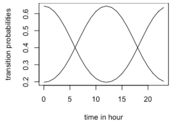
Preliminary Results

Simul Scenario	Setting	Model (AIC)	number of hidden states selected		
			2 (%)	3 (%)	4 (%)
No misspecification 	100 Obs, 10 Rep	Null	97	3	-
		mix 2	95	5	-
		TMBnorm	56	42	2
		TMBt	90	-	10
Mixture with 2 components 	100 Obs, 10 Rep	Null	60	38	2
		mix 2	82	18	-
		TMBnorm	49	51	-
		TMBt	95	-	5
Temporal variation 	100 Obs, 10 Rep	Null	97	3	0
		mix 2	86	14	-
		TMBnorm	51	49	-
		TMBt	90	3	7

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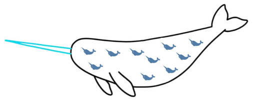
Preliminary Results

Simul Scenario	Setting	Model (AIC)	number of hidden states selected 2 (%)	3 (%)	4 (%)
No misspecification 	100 Obs, 10 Rep	Null	97	3	-
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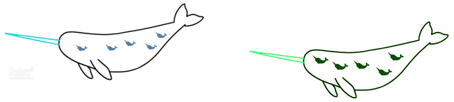
Mixture with 2 components 	100 Obs, 10 Rep	Null	60	38	2
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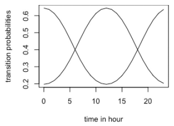
Temporal variation 	100 Obs, 10 Rep	Null	97	3	0
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		TMBnorm	51	49	-
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Fisheries and Oceans Canada

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***The Community of* Mittimatalik (Pond Inlet)**

WWF

Thank you

UBC

Nunavut Wildlife Management Board

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